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Spatial distribution of Joule heat in nanostructures

V L Gurevich and M I Muradov

A F Ioffe Institute, Russian Academy of Sciences, 194021 Saint Petersburg, Russia

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Abstract

The Joule heat generation in a quantum semiconductor nanowire joining two classical reservoirs is considered. We assume that the conductance of the system is determined by the phonon-assisted ballistic resistance. The spatial distribution of the generated heat is analysed. The heat generated within the nanostructure is determined. It is simply related to the phonon-assisted variation ΔG of the conductance. For a fixed voltage, the corresponding variation of the overall heat generation is negative and is determined by ΔG . Due to the same interaction the reduction of the heat generated in each reservoir is the same as the heat generated in the nanostructure. In other words, the presence of collisions within a nanowire does not violate the equality of the heat release in the reservoirs that are connected by the nanowire, although the rates and mechanisms of relaxation there may be different. We come to the conclusion that further investigation of various situations where the nanostructures are involved is needed to gain understanding as to why in some cases different heat generation in the two reservoirs is observed and predicted.

1. Introduction

During recent years various properties of quantum point contacts (such as quantum wires) have been investigated theoretically and by experiment. Among them one can name a step-like variation of conductance (ohmic and non-ohmic), shot noise, ultrasonic absorption, a number of optical phenomena and various other transport effects.

The resistance of a classical ballistic point contact between two metals was considered by Sharvin [1]. The characteristic dimensions of the contact were assumed to be much larger than the de Broglie wavelength. The quantization of the conductance of quantum ballistic contacts in units of $G_0 = e^2/\pi\hbar$ has been observed in [2, 3]. The spatial distribution of an electrostatic potential in quantum transport has been investigated for many different shapes of the contacts (see, e.g. [4, 5] and references therein).

Kulik *et al* [6] pointed out that the processes leading to electric resistance and heat generation were spatially separated in a classical point contact. Thermoelectric properties of quantum nanostructures have been studied quite extensively [7–12]. It has been shown that due to the subband structure of a quantum point contact the oscillatory behaviour of

the thermopower, the thermal conductance and the Peltier coefficient can be observed [8–12]. In [12] quantum size effects of the kinetic coefficients have been observed in a quantum point contact and a usage of this structure as an effective local electron gas thermometer has been demonstrated.

The total amount of Joule heat generated by a quantum contact can be determined by simple energy conservation considerations provided that the total contact conductance G is known. However, the *spatial distribution* of the heat that can be expressed via the rate of entropy production should be treated specially for every particular physical situation.

The problem of heat distribution in nanostructures and at their contacts (reservoirs) is important from a theoretical point of view. It may also be of pivotal importance to the issue of proper operation of scaled-down devices and very large scale integration circuits. This problem has been also considered with regard to experiments [13, 14]. The spatial distribution of Joule heat generated by a current across such a point contact has been treated in the diffusive limit by Rokni and Levinson [15]. One of the results of this paper is that in general the Joule heat released in each of the two reservoirs is different; however, it can be the same in the case where the reservoirs are identical.

One of the results of the papers [16, 17] where the case of ballistic nanowires is considered is the conclusion that all the Joule heat is released in the reservoirs, and irrespective of the actual form and the rate of electron relaxation in both reservoirs, the amount of the heat released in each of them is the same. In the present paper we demonstrate that the reason is a special type of symmetry that exists in degenerate electron systems. There are, however, some differences between the situations considered in [15] and [17]. One point is that whereas the electron collisions are considered in [15] no collisions are assumed in the nanowire in [16, 17]. The purpose of the present paper is to investigate the heat generation under *qualitatively different conditions*, namely taking into account the electron–phonon collisions in a current-carrying semiconductor quantum nanostructures. In other words, we consider a regime where a pure ballistic conductance is violated by the electron–phonon collisions and would like to find out *whether the collisions in a spatially homogeneous nanowire can be responsible for the asymmetry of the heat release in the reservoirs*.

The electron–phonon scattering we consider in the present paper is rather weak. This means that the variation $|\Delta G|$ of the conductance due to the electron–phonon scattering is smaller than G . At the same time it is quite discernible in experiments, as detailed investigation of its temperature dependence and the estimates have shown in [18, 19]. The absolute value of $|\Delta G|/G_0$ can be as big as 1: see [19] and the estimation given in section 6. However, in view of the nonlocal character of the quasiballistic transport it is not obvious without calculation that the heat release within the nanostructure is also determined by ΔG .

One of the most interesting and important points of this investigation is to find out whether the electron–phonon collisions can in principle change the situation with regard to equality of the amount of heat released in both reservoirs irrespective to their form. It has been emphasized in [15] that one of the reasons for enhancement of the interest in the Joule heat is the seeming asymmetry of a point contact. It has been argued, erroneously, that, since the number of the electrons moving in one direction is prevailing, more heat should be released there. This reasoning is correct with regard to the reversible Peltier heat but not the Joule heat. It appears, as we will show, that the equality of the Joule heat release in both reservoirs remains valid also provided a weak electron–phonon scattering is present in the nanowire. We analyse a *specific electron–hole symmetry* that is responsible for such a property provided the conduction electrons obey the Fermi statistics. As a result, we arrive at the qualitative conclusion that the collisions within the nanowire cannot be the single reason for the heat release asymmetry in the reservoirs, and provide a physical analysis of the situation.

There is another important point that is discussed in the present paper. The Joule heat generated *within the nanowire* that we calculate is due to the electron–phonon scattering. Sometimes an opinion is expressed that this part of the heat amounts to the energy transferred to the phonon system in the course of electron–phonon collisions. And sometimes such a conclusion may be true (cf [15]), but in the system considered in the present paper the physics is entirely different. Although energy relaxation is essential for the applicability of our method of the heat release calculation, in sections 3 and 6 we show that the heat release within the nanowire is due to the relaxation of the nonequilibrium part of the electron distribution function. The amount of heat directly associated with the nonequilibrium phonon generation is relatively small.

The paper is organized as follows. In section 2 we analyse the notion of mechanical energy, its dissipation and its relation with the entropy and heat production. In section 3 we consider the heat released in a nanowire. In order to calculate the heat in the almost ballistic nanowire it is sufficient to know the electron distribution function only in the zeroth approximation in the electron–phonon interaction since the equation for the entropy production already involves the electron–phonon scattering probability. To calculate the heat release in the reservoirs the iterative procedure is not sufficient and one needs an essentially different approach.

In section 4 we explicitly take into account the scattering processes in the reservoirs and determine the nonequilibrium deviation of the distribution function. In section 5 a special type of symmetry characteristic for degenerate electron systems is considered. This symmetry is responsible for the equality of the heat release in both reservoirs.

To gain a better understanding of the role of electron–phonon scattering in the heat generation in a nanowire we discuss in some detail the role of elastic and inelastic processes in section 6. This helps in understanding the physics of heat generation due to the electron–phonon collisions in a nanowire.

2. Mechanical energy and heat

It will be convenient to consider an isolated system. Therefore we will have in mind the following physical situation. There is a capacitor which is discharged through the conductor of interest. The product RC of the whole system, R and C being the resistance and capacitance respectively, is much bigger than any relaxation time characterizing the electron or phonon system of the conductor. This means that for all practical purposes the conduction process can be looked upon as a stationary one. The total energy of the system, \mathcal{U} , is conserved while its total entropy, \hat{S} , is growing. The rate of heat generation is expressed through $T\partial\hat{S}/\partial t$, where T is the temperature. So our main purpose will be to calculate the rate of entropy production.

Although we are mostly interested in the consideration of microstructures, to calculate the heat production we will also need the Boltzmann equation for bulk samples, since an essential part of the heat is released in the reservoir regions. In what follows we will briefly discuss the properties of the Boltzmann equation, bringing it to such a form that can be used in the subsequent sections.

The Boltzmann equation for the electron distribution function, $F_{\mathbf{p}}(\mathbf{r}, t)$, is

$$\frac{\partial F_{\mathbf{p}}}{\partial t} + \frac{\partial \tilde{\epsilon}_{\mathbf{p}}}{\partial \mathbf{p}} \frac{\partial F_{\mathbf{p}}}{\partial \mathbf{r}} - \frac{\partial \tilde{\epsilon}_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\partial F_{\mathbf{p}}}{\partial \mathbf{p}} = \left[\frac{\partial F_{\mathbf{p}}}{\partial t} \right]_{\text{coll}}. \quad (2.1)$$

Here $\tilde{\epsilon}_{\mathbf{p}}(\mathbf{r}) = \epsilon_{\mathbf{p}} + e\phi(\mathbf{r})$ is the total energy of an electron, and $\phi(\mathbf{r})$ is the electrostatic potential. The term on the right-hand side describes the collisions of electrons with impurities (elastic collisions) and with phonons (inelastic collisions). The latter is expressed through the phonon

distribution function $N_{\mathbf{q}}$, which satisfies the Boltzmann equation for phonons:

$$\frac{\partial N_{\mathbf{q}}}{\partial t} + \frac{\partial \Omega_{\mathbf{q}}}{\partial \mathbf{q}} \frac{\partial N_{\mathbf{q}}}{\partial \mathbf{r}} - \frac{\partial \Omega_{\mathbf{q}}}{\partial \mathbf{r}} \frac{\partial N_{\mathbf{q}}}{\partial \mathbf{q}} = \left[\frac{\partial N_{\mathbf{q}}}{\partial t} \right]_{\text{coll}}. \quad (2.2)$$

Here $\Omega_{\mathbf{q}}$ is the phonon frequency (which in general can be a smooth function of the spatial coordinates). The term on the right-hand side describes the collisions of phonons with impurities (elastic collisions), with electrons and phonon–phonon collisions due to the anharmonicity. One can also include in $[\partial F_{\mathbf{p}}/\partial t]_{\text{coll}}$ of equation (2.1) a term describing electron–electron collisions.

Now we will discuss the energy conservation, or, in other words, the overall rate of heat release. The same method can be applied to the Boltzmann equation describing transport in nanostructures (see equation (3.3) below). We multiply equation (2.1) by $\epsilon_{\mathbf{p}}(\mathbf{r}) + e\phi(\mathbf{r})$, integrate by $d\xi_{\mathbf{p}}$, where $d\xi_{\mathbf{p}} = d^3 p/(2\pi\hbar)^3$ (the summation over the spin variable is implied here; we assume that the electron energy is spin-independent), and add equation (2.2), transforming it in the same manner as equation (2.1) (see below). Further on we make use of the following property of collision term of the Boltzmann equation:

$$\int d\xi_{\mathbf{p}} \left[\frac{\partial F_{\mathbf{p}}}{\partial t} \right]_{\text{coll}} = 0, \quad (2.3)$$

which means that the electron collisions conserve the number of electrons, and the charge conservation law that follows from the Boltzmann equation:

$$e \frac{\partial n}{\partial t} + \text{div } \mathbf{j} = 0, \quad (2.4)$$

where

$$n(\mathbf{r}) = \int d\xi_{\mathbf{p}} F_{\mathbf{p}}(\mathbf{r}), \quad \mathbf{j} = e \int d\xi_{\mathbf{p}} F_{\mathbf{p}}(\mathbf{r}) \frac{\partial \epsilon}{\partial \mathbf{p}} \quad (2.5)$$

are the electron concentration and the current density respectively. One can write the resulting energy conservation equation in the following form:

$$\frac{\partial U}{\partial t} + \text{div } \mathbf{Q} = \mathbf{j} \mathbf{E}, \quad (2.6)$$

where $\mathbf{E} = -\nabla\phi$ is the electric field. Here we made use of the fact that if electron–phonon collisions are taken into account the total energy of the electron and phonon systems is conserved. As a result, one obtains equation (2.6), where the energy density and the energy flux are given by

$$U = \int d\xi_{\mathbf{p}} \epsilon_{\mathbf{p}} F_{\mathbf{p}} + \int d\eta_{\mathbf{q}} \hbar \Omega_{\mathbf{q}} N_{\mathbf{q}}, \quad (2.7)$$

$$\mathbf{Q} = \int d\xi_{\mathbf{p}} \epsilon_{\mathbf{p}} \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} F_{\mathbf{p}} + \int d\eta_{\mathbf{q}} \hbar \Omega_{\mathbf{q}} N_{\mathbf{q}} \frac{\partial \Omega_{\mathbf{q}}}{\partial \mathbf{q}}. \quad (2.8)$$

Here $d\eta_{\mathbf{q}} = d^3 q/(2\pi)^3$; the summation over the phonon branches is implied.

In the derivation of equation (2.6) we made use of the following transformation:

$$\int d\xi_{\mathbf{p}} \epsilon_{\mathbf{p}} \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \frac{\partial F_{\mathbf{p}}}{\partial \mathbf{r}} - \int d\xi_{\mathbf{p}} \epsilon_{\mathbf{p}} \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\partial F_{\mathbf{p}}}{\partial \mathbf{p}} = \frac{1}{2} \int d\xi_{\mathbf{p}} \left[\frac{\partial}{\partial \mathbf{p}} \left(\epsilon_{\mathbf{p}}^2 \frac{\partial F_{\mathbf{p}}}{\partial \mathbf{r}} \right) - \frac{\partial}{\partial \mathbf{r}} \left(\epsilon_{\mathbf{p}}^2 \frac{\partial F_{\mathbf{p}}}{\partial \mathbf{p}} \right) \right]. \quad (2.9)$$

The first term on the right-hand side can be transformed into the integral over the surface of the first Brillouin zone. One can see that the integrands over the opposite faces of the first Brillouin zone have the same absolute values but opposite signs. Therefore the whole integral vanishes. The same transformation has been applied to the phonon variables.

The equations for heat production given in the present section will be used below for the calculation of the spatial distribution of generated heat. This applies both to the bulk samples and the nanostructures. Indeed, (as we will see) a nonballistic current conduction through a narrow nanostructure may result in the heat release in the nanostructure itself and the adjoining bulk reservoirs.

The equation for the total entropy of the electron and phonon gases reads

$$\hat{S} = \int d^3r S(\mathbf{r}). \quad (2.10)$$

$S = S^{(e)} + S^{(p)}$, where (we will set $k_B = 1$ throughout the paper)

$$S^{(e)} = \int d\xi_{\mathbf{p}} s_{\mathbf{p}}^{(e)}, \quad s_{\mathbf{p}}^{(e)} = -[F_{\mathbf{p}} \ln F_{\mathbf{p}} + (1 - F_{\mathbf{p}}) \ln(1 - F_{\mathbf{p}})], \quad (2.11)$$

$$S^{(p)} = \int d\eta_{\mathbf{q}} s_{\mathbf{q}}^{(p)}, \quad s_{\mathbf{q}}^{(p)} = [-N_{\mathbf{q}} \ln N_{\mathbf{q}} + (1 + N_{\mathbf{q}}) \ln(1 + N_{\mathbf{q}})]. \quad (2.12)$$

Differentiating S with respect to time and inserting $\partial F_{\mathbf{p}}/\partial t$ and $\partial N_{\mathbf{q}}/\partial t$ from the Boltzmann equations for the electrons and phonons, equations (2.1) and (2.2), we obtain

$$\frac{\partial S}{\partial t} + \text{div } \mathbf{s} = \left[\frac{\partial S}{\partial t} \right]_{\text{coll}} \quad (2.13)$$

and

$$\frac{d\hat{S}}{dt} = \int d^3r \left[\frac{\partial S}{\partial t} \right]_{\text{coll}}. \quad (2.14)$$

Here the rate of entropy density production is given by

$$\left[\frac{\partial S}{\partial t} \right]_{\text{coll}} = \int d\xi_{\mathbf{p}} \left[\frac{\partial F_{\mathbf{p}}}{\partial t} \right]_{\text{coll}} \ln \frac{1 - F_{\mathbf{p}}}{F_{\mathbf{p}}} + \int d\eta_{\mathbf{q}} \left[\frac{\partial N_{\mathbf{q}}}{\partial t} \right]_{\text{coll}} \ln \frac{1 + N_{\mathbf{q}}}{N_{\mathbf{q}}}, \quad (2.15)$$

while $\mathbf{s} = \mathbf{s}^{(e)} + \mathbf{s}^{(p)}$ is the entropy flux. We have

$$\mathbf{s}^{(e)} = \int d\xi_{\mathbf{p}} F_{\mathbf{p}} \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \ln \frac{1 - F_{\mathbf{p}}}{F_{\mathbf{p}}}, \quad \mathbf{s}^{(p)} = \int d\eta_{\mathbf{q}} N_{\mathbf{q}} \frac{\partial \Omega_{\mathbf{q}}}{\partial \mathbf{q}} \ln \frac{1 + N_{\mathbf{q}}}{N_{\mathbf{q}}}. \quad (2.16)$$

In deriving equation (2.13) we have taken into account that the integrals

$$\int d\xi_{\mathbf{p}} \frac{\partial}{\partial \mathbf{p}} \left(\ln \frac{1 - F_{\mathbf{p}}}{F_{\mathbf{p}}} \frac{\partial \tilde{\epsilon}_{\mathbf{p}}}{\partial \mathbf{r}} F_{\mathbf{p}} \right) \quad \text{and} \quad \int d\eta_{\mathbf{q}} \frac{\partial}{\partial \mathbf{q}} \left(\ln \frac{1 + N_{\mathbf{q}}}{N_{\mathbf{q}}} \frac{\partial \Omega_{\mathbf{q}}}{\partial \mathbf{r}} N_{\mathbf{q}} \right) \quad (2.17)$$

vanish due to the periodicity of the integrands in reciprocal lattice space. We have also assumed that the entropy flux through the sample's surface vanishes.

It is important that the quantity (2.15) is within the accepted approximation a *local quantity*, depending on the space coordinates as parameters. This enables one to separate the problems of the heat release in the quantum wire (or, in other words, in the squeezed part of the point contact) and in the reservoirs that we are going to solve in the present paper. The Boltzmann equation for a quantum wire as well as the corresponding rate of the entropy variation will be formulated below in section 3.

A univocal definition of the heat released by a physical system irrespective of whether it is in an equilibrium state, or in contact with a thermostat, or in a nonequilibrium state, can be found for instance in [20 sections 14 and 20]. Using this definition, one can state that an isolated macroscopic system possesses a *mechanical energy*, \mathcal{E} . This means that, owing to internal processes, without a resulting variation of its volume, the system can execute work on external systems. In general the amount of work depends on how the internal state of the

system under consideration is changed. Mechanical energy is the maximal amount of work that can be produced if the system finally reaches the state of thermodynamic equilibrium.

It is well known (see [20]) that the rate of heat generation $dQ/dt = -d\mathcal{E}/dt$ equals the negative rate of mechanical energy dissipation

$$\frac{dQ}{dt} = -\frac{d\mathcal{E}}{dt} = \frac{d\mathcal{U}}{d\hat{S}} \frac{d\hat{S}}{dt} = T_0 \frac{d\hat{S}}{dt}, \quad (2.18)$$

where $\mathcal{U}(\hat{S})$ is the total energy of the system expressed as a function of its entropy \hat{S} by the relations of equilibrium thermodynamics, T_0 being the temperature of such an equilibrium system whose entropy is equal to \hat{S} . (In general T_0 is not a real temperature of some system or a thermostat but rather an ancillary quantity that can be introduced irrespective of whether the system in consideration is in equilibrium or not, including the nonequilibrium states that cannot be described by introduction of any temperature.) Making use of equation (2.14), one gets in our case

$$\frac{dQ}{dt} = T_0 \frac{d\hat{S}}{dt} = T_0 \int d^3r \left[\frac{\partial S}{\partial t} \right]_{\text{coll}}. \quad (2.19)$$

This means that *the mechanical energy* of an isolated system, unlike its total energy, *is not conserved*. Due to the relaxation processes the mechanical energy of the system *dissipates into heat*. Equation (2.19) describes the rate of mechanical energy dissipation in the nonequilibrium system in consideration or, in other words, the rate of Joule heat generation both in the nanowire and in the reservoirs.

We are dealing with two large reservoirs connected by a nanowire. Before having been connected to a source of voltage the whole system is in the equilibrium and can be described by a common temperature. Now we connect the system with a battery that results in shifts of the chemical potentials of the reservoirs. The reservoirs have the same temperature and begin to exchange particles and energy with each other. As, however, the current across the nanowire is very weak one can assume that the reservoirs maintain equilibrium with the same temperature although their chemical potentials differ. The electron distribution function in the nanowire is a nonequilibrium one, and we do not ascribe any temperature to the electron distribution in the wire. The temperature T in equation (3.8) below follows directly from the definition (2.19).

3. Heat release in nanostructure

Consider a nanowire in which the x -axis is the direction along which the electron motion is infinite and current flows. Along the perpendicular direction(s) the electron motion is quantized. To be definite we will discuss a three-dimensional (3D) case although one can turn to a 2D case by a simple change of notation. We will consider the so-called adiabatic constriction (see Glazman *et al* [21]) where the potential profile varies smoothly along the x -axis on the scale of the electron de Broglie wavelength. We assume that the electron mean free path is bigger than the characteristic dimensions of the microstructure. Then there is a system of 1D electron bands (*channels*) describing the electrons' motion in the x -direction both in the microstructure and in the adjoining parts of the contacts. This motion will be considered (quasi)classically. The minimum width of the constriction is of the order of the de Broglie wavelength; therefore the transverse motion is quantized. In the spirit of an approach of [21] we assume that the variables x and \mathbf{r}_\perp are separable in the adiabatic approximation. This means that for each value of x the \mathbf{r}_\perp -dependence of the potential determines the wavefunction of transverse quantization $\eta_n(\mathbf{r}_\perp; x)$ that depends on x as a parameter. Here n is the quantum number of transverse quantization. In other words, the distribution function is defined as a

coarse-grained function of the longitudinal quasimomentum and coordinate (only for such a function can one introduce the coordinate x and the x -component of the (quasi)momentum simultaneously). Therefore such a distribution function cannot describe such a phenomena as Friedel-like oscillations. They are smeared out due to the averaging as $\int_{-\Delta x/2}^{\Delta x/2} \sin 2k_F(x + \Delta x)$, where $|\Delta x| \gg 1/k_F$.

The electron spectrum depends on x , and this dependence has the form

$$\epsilon_n(p; x) = p^2/2m + \epsilon_n(0; x), \quad (3.1)$$

where m is the electron effective mass and $\epsilon_n(0, x)$ is the position of the band bottom that depends on x as a parameter. $\epsilon_n(0; x)$ is the solution of the eigenvalue problem for the Hamiltonian

$$H = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + e\Phi(\mathbf{r}_\perp; x). \quad (3.2)$$

Here the potential $\Phi(\mathbf{r}_\perp; x)$ can be regarded as including both the constriction potential profile and the self-consistent field.

Now, $d\xi_p = dp/2\pi\hbar$ (summation over the spin variable is implied), $v_p = \partial\epsilon_{np}/\partial p = p/m$ is the electron group velocity (that does not depend explicitly on n), and $F_{np}(x)$ is the electron distribution function that depends on the quantum number n as a parameter, while p (the x -component of the electron quasimomentum) and x can be considered as classical variables. The distribution function $F_{np}(x)$ satisfies the Boltzmann equation

$$v \frac{\partial F_{np}(x)}{\partial x} - e \frac{\partial \varphi_n(x)}{\partial x} \frac{\partial F_{np}(x)}{\partial p} = \left[\frac{\partial F_{np}}{\partial t} \right]_{\text{coll}}. \quad (3.3)$$

Here the potential averaged over the transverse coordinates

$$\varphi_n(x) = \int d\mathbf{r}_\perp |\eta_n(\mathbf{r}_\perp; x)|^2 \varphi(\mathbf{r}) \quad (3.4)$$

tends to the external fixed value $+V/2$ far in the left contact and to the value $-V/2$ far in the right contact, and

$$\begin{aligned} \left[\frac{\partial F_{np}}{\partial t} \right]_{\text{coll}} &= \sum_{n'} \int d\xi_{p'} \int d\eta_{\mathbf{q}} \{ w_{nn'}(p, p'; \mathbf{q}) \\ &\times [F_{n'p'}(1 - F_{np})(N_{\mathbf{q}} + 1) - F_{np}(1 - F_{n'p'})N_{\mathbf{q}}] \\ &+ w_{n'n}(p', p; \mathbf{q}) [F_{n'p'}(1 - F_{np})N_{\mathbf{q}} - F_{np}(1 - F_{n'p'})(N_{\mathbf{q}} + 1)] \}. \end{aligned} \quad (3.5)$$

Here the phonon distribution function $N_{\mathbf{q}}$ is to be determined from the corresponding Boltzmann equation and

$$w_{nn'}(p, p'; \mathbf{q}) \propto \delta_{p', p+\hbar q_x} \delta(\epsilon_{n'p'} - \epsilon_{np} - \hbar\Omega_{\mathbf{q}}) \quad (3.6)$$

is the probability of an electron transition from the state specified by the quantum numbers n' , p' to the state n , p accompanied by emission of a phonon with the wavevector \mathbf{q} . In such a general form this equation is valid for any phonon state. If one considers the interaction of the electrons of a nanowire with the bulk phonons then the perturbation theory gives (cf [18, 19])

$$w_{nn'}(p, p', \mathbf{q}) = \frac{2\pi}{\hbar} |c_{\mathbf{q}}|^2 | \langle n | \exp(i\mathbf{q}_\perp \mathbf{r}_\perp) | n' \rangle |^2 \delta[\epsilon_{np} - \epsilon_{n'p'} + \hbar\Omega_{\mathbf{q}}] \delta_{p', p+\hbar q_x}. \quad (3.7)$$

Here $c_{\mathbf{q}}$ is the matrix element of the interaction of electrons with bulk phonons; \mathbf{r}_\perp denotes y, z , while \mathbf{q}_\perp denotes q_y, q_z . We assume that the nonequilibrium phonons are removed from the vicinity of the quantum wire, so that their distribution function is the equilibrium Bose function.

The rate of heat release for a nanostructure system is given by

$$\frac{dQ}{dt} = T \frac{d\hat{S}}{dt}. \quad (3.8)$$

For an electron system interacting with phonons the entropy production is given by

$$\frac{d\hat{S}}{dt} = \int dx \left[\frac{\partial S}{\partial t} \right]_{\text{coll}}, \quad (3.9)$$

where

$$\begin{aligned} \left[\frac{\partial S}{\partial t} \right]_{\text{coll}} = & \sum_{nn'} \int d\xi_p \int d\xi_{p'} \int d\eta_{\mathbf{q}} w_{nn'}(p, p'; \mathbf{q}) \ln \frac{F_{n'p'}(1 - F_{np})(N_{\mathbf{q}} + 1)}{F_{np}(1 - F_{n'p'})N_{\mathbf{q}}} \\ & \times [F_{n'p'}(1 - F_{np})(N_{\mathbf{q}} + 1) - F_{np}(1 - F_{n'p'})N_{\mathbf{q}}]. \end{aligned} \quad (3.10)$$

Following the approach developed by Landauer [22], Imry [23], and Büttiker [24], one can present the nonequilibrium electron distribution function in the nanowire in the zeroth approximation in the electron–phonon scattering in the form

$$F_{np0} = \theta(p)F^{(0)}(\varepsilon_{np} - \mu^{(+)}) + \theta(-p)F^{(0)}(\varepsilon_{np} - \mu^{(-)}), \quad (3.11)$$

where $F^{(0)}$ is the Fermi function, $\mu^{(\pm)} = \mu \pm eV/2$. To obtain this solution of equation (3.3) we neglect the collision term and assume that there is no field within the nanowire. This equation describes only the transmitted modes, i.e. we consider the simplest possible situation. We have not introduced here transmission and reflection coefficients implying that any mode having the energy greater (smaller) than some maximum energy is totally transmitted (reflected). Inserting this function in equation (3.10) and using the equilibrium phonon function $N_{\mathbf{q}}$ one can see that only the domains $pp' < 0$ contribute to the integral over the momenta. Consider first the $p > 0, p' < 0$ domain. Using the identities

$$1 - F^{(0)}(\varepsilon) = e^{\varepsilon/T} F^{(0)}(\varepsilon), \quad 1 + N_{\mathbf{q}}^{(0)} = e^{\hbar\Omega_{\mathbf{q}}/T} N_{\mathbf{q}}^{(0)}$$

and the energy conservation laws imposed by the δ -function in the expression for the transition probability, we can sufficiently simplify the integrand: the logarithmic term in the integrand can be reduced to $-eV/T$, while the difference between the distribution function product yields

$$\begin{aligned} & F_{n'p'}(1 - F_{np})(N_{\mathbf{q}} + 1) - F_{np}(1 - F_{n'p'})N_{\mathbf{q}} \\ & = -2 \sinh\left(\frac{eV}{2T}\right) N_{\mathbf{q}} F^{(0)}(\varepsilon_{np} - \mu^{(+)}) F^{(0)}(\varepsilon_{n'p'} - \mu^{(-)}) \\ & \quad \times \exp[(\varepsilon_{np} + \hbar\Omega_{\mathbf{q}} - \mu)/T]. \end{aligned} \quad (3.12)$$

For $p < 0, p' > 0$ we get the same expressions but with V replaced by $-V$. Discarding $eV/2$ in the arguments of the Fermi functions and replacing $\sinh(eV/2T)$ by $eV/2T$ in the linear response regime, we arrive at

$$\begin{aligned} \left[\frac{\partial S}{\partial t} \right]_{\text{coll}} = & 2 \left(\frac{eV}{T} \right)^2 \sum_{nn'} \int_0^\infty d\xi_p \int_0^\infty d\xi_{p'} \int d\eta_{\mathbf{q}_\perp} w_{nn'}(p, p'; \mathbf{q}) N_{\mathbf{q}} \\ & \times F^{(0)}(\varepsilon - \mu)[1 - F^{(0)}(\varepsilon' - \mu)]. \end{aligned} \quad (3.13)$$

Here in the transition probability due to the quasimomentum conservation along the longitudinal direction (x -axis), $q_x = (p + p')/\hbar$, $\varepsilon = \varepsilon_{np}$ and $\varepsilon' = \varepsilon_{np} + \hbar\Omega_{\mathbf{q}}$. Introducing the phonon collision-controlled part ΔG of the conductance

$$\Delta G = -2 \frac{e^2 L}{T} \sum_{nn'} \int d\eta_{\mathbf{q}_\perp} |\langle n' | e^{i\mathbf{q}_\perp \cdot \mathbf{r}_\perp} | n \rangle|^2 C_{n'n}, \quad (3.14)$$

$$C_{n'n} = \int_0^\infty d\xi_p \int_0^\infty d\xi_{p'} N_{\mathbf{q}} \frac{2\pi}{\hbar} |c_{\mathbf{q}}|^2 \delta(\varepsilon' - \varepsilon - \hbar\Omega_{\mathbf{q}}) [1 - F^{(0)}(\varepsilon' - \mu)] F^{(0)}(\varepsilon - \mu) \quad (3.15)$$

(where L is the length of the nanowire), we can write for the entropy production

$$\left[\frac{\partial S}{\partial t} \right]_{\text{coll}} = -\frac{V^2}{LT} \Delta G, \quad (3.16)$$

whereas for the heat generation within the nanowire we get

$$\left(\frac{dQ}{dt} \right)_{\text{nan}} = T \int_{-L/2}^{L/2} dx \left[\frac{\partial S}{\partial t} \right]_{\text{coll}} = -V^2 \Delta G. \quad (3.17)$$

For a uniform quantum wire the heat is generated uniformly along the whole length L of the wire. We assume that the radius of curvature of the parts that join the nanowire with the reservoirs is much smaller than L . The change of the conductance due to electron–phonon collisions ΔG being negative, the corresponding contribution to the heat release within the nanowire at the fixed applied voltage is naturally positive, whereas the total conductance decreases. This may seem strange since provided the applied voltage is fixed the Joule heat $V^2 G$ must decrease with decrease of the conductance while our formula indicates a positive contribution to the heat. However, this is quite natural since the electron scattering changes the electron distribution function both in the nanowire and in the reservoirs. In the next section we will show that the decrease of the heat released in the reservoirs turns out to be twice as big as the heat released within the wire.

4. Electron distribution function in nanowire

We assume the nanowire to be a linear structure of length L . The situation in the nanowire can be described by iterations treating the electron–phonon collision term as a perturbation. We have for the distribution function

$$F_{np}(x) = F_{np0} + \Delta F_{np}(x), \quad (4.1)$$

where F_{np0} is given by equation (3.11) and $\Delta F_{np}(x)$ satisfies the Boltzmann equation

$$v \frac{\partial \Delta F_{np}(x)}{\partial x} - e \frac{\partial \Delta \varphi_n(x)}{\partial x} \frac{\partial F_{np0}}{\partial p} = I[F_{np0}]. \quad (4.2)$$

Here $\Delta \varphi_n(x)$ is the variation of the self-consistent potential $\varphi_n(x)$ (introduced in equation (3.4) as an average with respect to transverse coordinates) due to the collisions. The solution F_{np0} obtained from equation (3.11) entirely neglecting the collisions is inserted into the scattering integral (for brevity we have introduced a more compact notation $I[F_{np0}]$ for the latter) as well as into the drift term. We integrate this equation along the electron trajectory. Therefore, the solution can be written as

$$\Delta F_{np}(x) = \left(x \pm \frac{L}{2} \right) \frac{1}{v} I[F_{np0}] + e \Delta \varphi_n \frac{\partial F_{np}^{(0)}}{\partial \varepsilon_{np}}, \quad (4.3)$$

where the upper (lower) sign in the first term stands for $p > 0$ ($p < 0$). The potential $\Delta \varphi_n(x)$ must in general be determined from the Poisson equation. If the screening length is much smaller than the dimensions of the wire it is equivalent to the neutrality condition. Then

$$e \Delta \varphi_n(x) = x \frac{1}{v_n} \int d\xi_p I[F_{np0}]/v, \quad (4.4)$$

where we have introduced the density of states for the n th channel,

$$v_n = \int d\xi_p \left(-\frac{\partial F_{np}^{(0)}}{\partial \varepsilon_{np}} \right). \quad (4.5)$$

Taking into account equations (3.11) and (4.3), we get the deviation of the distribution function of electrons moving to the right (at $x = L/2$):

$$\Delta F_{np}(L/2) = \left(-\frac{\partial F_{np}^{(0)}}{\partial \varepsilon_{np}} \right) \left(eV - \frac{L}{2v_n} \int d\xi_p \frac{1}{v} I[F_{np0}] \right) + \frac{L}{v} I[F_{np0}]. \quad (4.6)$$

For the electrons moving to the left (i.e. for $p < 0$) at the same point one gets

$$\Delta F_{np}(L/2) = \left(-\frac{\partial F_{np}^{(0)}}{\partial \varepsilon_{np}} \right) \left(-\frac{L}{2v_n} \int d\xi_p \frac{1}{v} I[F_{np0}] \right). \quad (4.7)$$

In the right contact the deviation ΔF_{np} of the distribution functions from the equilibrium satisfies the Boltzmann equation

$$v \frac{\partial \Delta F_{np}}{\partial x} - e \frac{\partial \varphi_n}{\partial x} \frac{\partial F_{np}^{(0)}}{\partial p} = I \Delta F_{np} \quad (4.8)$$

with the boundary condition at $L/2$ represented by equation (4.6). Here I is the linearized integral operator (3.5) acting on ΔF_{np} . Its explicit expression for the electron–phonon collisions is

$$\begin{aligned} I \Delta F_{np} = & \sum_{n'} \int d\xi_{p'} \int d\eta_{\mathbf{q}} \Delta F_{n'p'} \{ w_{nn'}(p, p'; \mathbf{q}) [(1 - F_{np}^{(0)}) N_{\mathbf{q}} + F_{np}^{(0)} (N_{\mathbf{q}} + 1)] \\ & + w_{n'n}(p', p; \mathbf{q}) [(1 - F_{np}^{(0)}) (N_{\mathbf{q}} + 1) + F_{np}^{(0)} N_{\mathbf{q}}] \\ & - \Delta F_{np} \sum_{n'} \int d\xi_{p'} \int d\eta_{\mathbf{q}} \{ w_{nn'}(p, p'; \mathbf{q}) [F_{n'p'}^{(0)} (N_{\mathbf{q}}^{(0)} + 1) + (1 - F_{n'p'}^{(0)}) N_{\mathbf{q}}^{(0)}] \\ & + w_{n'n}(p', p; \mathbf{q}) [F_{n'p'}^{(0)} N_{\mathbf{q}}^{(0)} + (1 - F_{n'p'}^{(0)}) (N_{\mathbf{q}}^{(0)} + 1)] \}. \end{aligned} \quad (4.9)$$

It is an algebraic sum of two terms. One of them is an integral term, where $\Delta F_{n'p'}$ is in the integrand. The other one is a time-of-relaxation term which has the form

$$\left[\frac{\partial F_{np}}{\partial t} \right]_{\text{coll}}^{(\tau)} = -\frac{\Delta F_{np}}{\tau_{np}}, \quad (4.10)$$

where the relaxation time τ_{np} is given by

$$\begin{aligned} \frac{1}{\tau_{np}} = & \sum_{n'} \int d\xi_{p'} \int d\eta_{\mathbf{q}} \{ w_{nn'}(p, p'; \mathbf{q}) [F_{n'p'}^{(0)} (N_{\mathbf{q}}^{(0)} + 1) + (1 - F_{n'p'}^{(0)}) N_{\mathbf{q}}^{(0)}] \\ & + w_{n'n}(p', p; \mathbf{q}) [F_{n'p'}^{(0)} N_{\mathbf{q}}^{(0)} + (1 - F_{n'p'}^{(0)}) (N_{\mathbf{q}}^{(0)} + 1)] \}. \end{aligned} \quad (4.11)$$

As indicated in [17], to describe the transport in the reservoirs the integral term can be discarded, so one can apply the relaxation time approximation. Thus equation (4.8) is reduced to

$$v \frac{\partial \Delta F_{np}}{\partial x} - e \frac{\partial \varphi_n}{\partial x} \frac{\partial F_{np}^{(0)}}{\partial p} = -\frac{\Delta F_{np}}{\tau_{np}}. \quad (4.12)$$

The solution for $p > 0, x > L/2$ can be written as

$$\begin{aligned} \Delta F_{np}(x) = & \Delta F_{np}(L/2) \exp[-(x - L/2)/v\tau_{np}] \\ & + \frac{\partial F_{np}^{(0)}}{\partial \varepsilon_{np}} \int_{L/2}^x dx' e^{\frac{\partial \varphi_n}{\partial x'}} \exp[-(x - x')/v\tau_{np}], \end{aligned} \quad (4.13)$$

whereas for $p < 0$ we have

$$\Delta F_{np}(x) = \frac{\partial F_{np}^{(0)}}{\partial \varepsilon_{np}} \int_{+\infty}^x dx' e^{\frac{\partial \varphi_n}{\partial x'}} \exp[-(x - x')/v\tau_{np}]. \quad (4.14)$$

Let us first concentrate on the distances $0 < x - L/2 \lesssim l$ (where l is the mean free path). In this region we can omit the second term in equation (4.13) and the expression given by equation (4.14). The omitted part of the distribution function is important only at distances from the constriction much greater than the mean free path. We will discuss this point in the next section.

As for equation (3.10) for the entropy production, in the same approximation it reduces to

$$\left[\frac{\partial S}{\partial t} \right]_{\text{coll}} = \sum_n \int d\xi_p \frac{(\Delta F_{np})^2}{\tau_{np} F_{np}^{(0)} (1 - F_{np}^{(0)})}. \quad (4.15)$$

In what follows we shall retain the terms proportional to V^2 and linear in the collision term. We have

$$\begin{aligned} [\Delta F_{np}(x)]^2 &= \theta(p) \left[-\frac{\partial F_{np}^{(0)}}{\partial \varepsilon_{np}} \right] eV \left\{ eV \left[-\frac{\partial F_{np}^{(0)}}{\partial \varepsilon_{np}} \right] + 2\frac{L}{v} I[F_{np0}] \right\} \\ &\times \exp[-2(x - L/2)/v\tau_{np}]. \end{aligned} \quad (4.16)$$

Inserting this expression into equation (4.15) and using the identity $F^{(0)}(1 - F^{(0)}) = -T \partial F^{(0)}/\partial \varepsilon$ we get

$$\begin{aligned} \left[\frac{\partial S}{\partial t} \right]_{\text{coll}} &= \sum_n \int_0^\infty d\xi_p \left(-\frac{\partial F_{np}^{(0)}}{\partial \varepsilon_{np}} \right) \frac{(eV)^2}{T \tau_{np}} \exp \left[\frac{-2(x - L/2)}{v\tau_{np}} \right] \\ &+ \sum_n \int_0^\infty d\xi_p \frac{2LeV}{T v \tau_{np}} I[F_{np0}] \exp \left[\frac{-2(x - L/2)}{v\tau_{np}} \right]. \end{aligned} \quad (4.17)$$

Here the first term reproduces the result for a purely ballistic nanowire (see [17]), while the second term is due to the electron–phonon contribution. Since

$$\sum_n \int_0^\infty d\xi_p I[F_{np0}] = \frac{V \Delta G}{eL}, \quad (4.18)$$

we have that the phonon contribution to the heat release in the right contact is given by

$$\left(\frac{dQ}{dt} \right)_{\text{ph}} = V^2 \Delta G. \quad (4.19)$$

Taking into account that in the left contact we have the same contribution, we arrive at the conclusion that the total negative contribution to the heat release due to electron–phonon interaction is $V^2 \Delta G$, as it should be from energy conservation considerations.

Now we will consider the region of relatively large values of x . Far enough from the constriction so that the width of the constriction is much greater than the de Broglie wavelength \hbar/\sqrt{mT} , one can use the classical Boltzmann equation describing particles with 3D momentum \mathbf{p} , and the situation can be described by the local conductivity σ . We assume that the classical description is possible at distances from the midpoint of our wire even smaller than the mean free path. It is seen from equations (4.13) and (4.14) that if $x - L/2 \gg l$ we have for the odd part of the distribution function

$$\Delta F_{np}(x) = eE_n(x)v\tau_{np} \left(-\frac{\partial F_{np}^{(0)}}{\partial \varepsilon_{np}} \right). \quad (4.20)$$

The very form of this expression makes it obvious that in the region of consideration we can write instead of this expression its 3D counterpart

$$\Delta F_{\mathbf{p}} = e\mathbf{E}v\tau_{\mathbf{p}} \left(-\frac{\partial F_{\mathbf{p}}^{(0)}}{\partial \varepsilon_{\mathbf{p}}} \right). \quad (4.21)$$

The continuity relation $\text{div } \mathbf{j} = 0$ then requires $\Delta\varphi(\mathbf{r}) = 0$. Imposing the constraint that the total current

$$\int \mathbf{j} d\mathbf{S} = - \int \sigma \nabla\varphi d\mathbf{S} = J \quad (4.22)$$

is equal to GV , we get that the potential far from the constriction in the right contact is described by

$$\varphi(\mathbf{r}) = -V \left(\frac{1}{2} - \frac{G}{\sigma\Omega r} \right), \quad (4.23)$$

where Ω is the solid angle describing the opening of the wire into the right contact.

Let us estimate the part of the heat released in this region and compare it with the heat released at distances of the order of the mean free path. Entropy production in this region is simply

$$T \left[\frac{\partial S}{\partial t} \right] = \sigma (\nabla\varphi)^2. \quad (4.24)$$

For the heat we have

$$\left(\frac{dQ}{dt} \right)^{(1)} = \sigma \int_{r>l} (\nabla\varphi)^2 dV = V^2 G \frac{G}{\Omega\sigma l}, \quad (4.25)$$

i.e. the heat release in this region is smaller according to the small parameter $G/\sigma l \sim (\lambda_F/l)^2$.

5. Electron–hole symmetry for Fermi-degenerate conductors

The property that the overall rate of heat generation in both reservoirs is the same is a consequence of a special symmetry that is typical for conductors where the electrons have a strong Fermi degeneracy. Now we will analyse this property in some detail.

To begin with, we replace F_{np} by

$$G_{np} = 1 - F_{np}. \quad (5.1)$$

G_{np} is the distribution function of holes. It may be useful to introduce such a function for the case of a strong Fermi degeneracy. Then one gets instead of equation (3.5)

$$\begin{aligned} \left[\frac{\partial G_{np}}{\partial t} \right]_{\text{coll}} = & \sum_{n'} \int d\xi_{p'} \int d\eta_{\mathbf{q}} \{ w_{nn'}(p, p'; \mathbf{q}) [G_{n'p'}(1 - G_{np})N_{\mathbf{q}} \\ & - G_{np}(1 - G_{n'p'})N_{\mathbf{q}} + 1] + w_{n'n}(p', p; \mathbf{q}) [G_{n'p'}(1 - G_{np})(N_{\mathbf{q}} + 1) \\ & - G_{np}(1 - G_{n'p'})N_{\mathbf{q}}] \}. \end{aligned} \quad (5.2)$$

Introducing the variable $y = -x$, one can write for a small nonequilibrium part ΔG

$$\frac{\partial \Delta G_{np}}{\partial y} + \frac{\Delta G_{np}}{\tau_{np}} = 0 \quad \text{for } p < 0. \quad (5.3)$$

This equation has the same form as equation (4.12) (if one neglects the potential term that gives, as we have seen, a small contribution). However, it describes the heat generation in the left reservoir. Accordingly, the value of the time of relaxation τ_{np} can be quite different as compared with that in the right reservoir provided the relaxation properties of the reservoirs are different. Integrating the solution of this equation over y , one can see that the amount of heat released in the contact per second is independent of the actual mechanism of relaxation. This is how the equivalence of the two reservoirs in regard to the heat generation can be obtained from the symmetry considerations. Physically it is immaterial as to whether one considers the heat

release as a result of transfer of electrons or holes. This is the reason for the equivalence of the heat release in both reservoirs.

We wish to emphasize that this symmetry is only approximate. It is the more exact the better the inequality

$$T/\mu^{(n)} \ll 1 \quad (5.4)$$

is fulfilled. The point is that the stripe of width T near the chemical potential $\mu^{(n)} \equiv \mu - \varepsilon_n(0; 0)$ is responsible for transport phenomena, including the heat generation. The electron–hole symmetry is valid provided one can neglect the variation of the density of electron states within this stripe. So the heat release in both reservoirs is the same, with the relative accuracy of $T/\mu^{(n)} \ll 1$.

6. Nonequilibrium phonons and their role

In the present section we will discuss the role of the phonons in the Joule heat balance due to the current in the quantum wire. Our purpose is to show that the energy transferred to the phonon system makes a small contribution to the total heat balance.

To begin with, we will analyse equation (3.14) for the contribution ΔG to the conductance due to the electron–phonon collisions. Let us consider the simplest case in which only the lowest electron miniband ($n=1$) is occupied. The quasimomentum transferred to a phonon is $q_x = (p+p')/\hbar \simeq 2p_F/\hbar$. We are interested in the case $sp_F \ll T$ (s is the sound velocity). In the opposite case in which $sp_F \gg T$, the number of phonons interacting with the conduction electrons and $|\Delta G|$ are small. This case is not interesting for us and we will not consider it.

Taking as an upper estimate for the matrix element $|\langle n | \exp(i\mathbf{q}_\perp \mathbf{r}_\perp) | n' \rangle|$ the quantity $1/(1+q_\perp^2 d^2)$ (d being the width of the quantum well) we see that only the phonons with transverse quasimomentum $q_\perp \sim 1/d$ contribute to ΔG .

Assuming $\hbar/d \leq p_F$ and restricting ourselves to the case $sp_F \ll T$ so that $\hbar\Omega_{\mathbf{q}} \ll T$, we can discard $\hbar\Omega_{\mathbf{q}}$ in the δ -function describing energy conservation and use $N_{\mathbf{q}} \simeq T/\hbar\Omega_{\mathbf{q}}$. The electron–phonon coupling constant $2\pi|c_{\mathbf{q}}|^2/\hbar$ for the deformation potential interaction can be written as $\pi\Lambda q^2/\rho\Omega_{\mathbf{q}}$, where Λ is the deformation potential constant, while ρ is the mass density. We get the following estimation for ΔG :

$$\Delta G \sim -\frac{e^2}{\pi\hbar} \frac{LT\Lambda^2}{\hbar^2 v_F^2 \rho s^2 d^2}. \quad (6.1)$$

As one knows the probabilities of electron–phonon collisions one can estimate the energy \dot{U}_{ph} transferred to the phonon system per second. This energy can be expressed through the phonon–electron scattering integral taking into account the terms $\hbar\Omega_{\mathbf{q}}$:

$$\left[\frac{\partial N_{\mathbf{q}}}{\partial t} \right]_{\text{coll}} = \sum_{nn'} \int d\xi_p \int d\xi_{p'} w_{nn'}(p, p'; \mathbf{q}) [F'(1-F)(N_{\mathbf{q}}+1) - F(1-F')N_{\mathbf{q}}]. \quad (6.2)$$

Here, considering the scattering as weak, one can use the equilibrium phonon distribution function and the electron distribution function of the zeroth order in the electron–phonon interaction (equation (3.11)). The integrand can then be simplified to

$$F'_0(1-F_0)(N_{\mathbf{q}}+1) - F_0(1-F'_0)N_{\mathbf{q}} = \pm 2 \sinh\left(\frac{eV}{2T}\right)(1+N_{\mathbf{q}}) \\ \times \exp[(\varepsilon_{np} - \mu)/T] F^{(0)}(\varepsilon_{np} - \mu \pm eV/2) F^{(0)}(\varepsilon_{n'p'} - \mu \mp eV/2) \quad (6.3)$$

for the domains $p < 0$, $p' > 0$ and $p > 0$, $p' < 0$ respectively. The sum of these contributions gives the difference between the electron distribution functions shifted by eV/T . Expanding

the difference, we get in the integrand

$$N_{\mathbf{q}} \left(\frac{eV}{2T} \right)^2 F^{(0)}(\varepsilon_{np} - \mu) [1 - F^{(0)}(\varepsilon_{n'p'} - \mu)] [F^{(0)}(\varepsilon_{np} - \mu) - F^{(0)}(\varepsilon_{n'p'} - \mu)]. \quad (6.4)$$

The rate of energy transfer to the phonon system can then be written as

$$\begin{aligned} \frac{dU_{\text{ph}}}{dt} &= \frac{e^2 V^2 L}{T} \sum_{nn'} \int d\eta_{\mathbf{q}\perp} |\langle n' | e^{i\mathbf{q}\perp \mathbf{r}\perp} | n \rangle|^2 B_{n'n}, \\ B_{n'n} &= \int_0^\infty d\xi_p \int_0^\infty d\xi_{p'} N_{\mathbf{q}} \frac{2\pi}{\hbar} |c_{\mathbf{q}}|^2 \delta(\varepsilon' - \varepsilon - \hbar\Omega_{\mathbf{q}}) \\ &\quad \times [1 - F^{(0)}(\varepsilon')] F^{(0)}(\varepsilon) \frac{\hbar\Omega_{\mathbf{q}}}{T} [F^{(0)}(\varepsilon) - F^{(0)}(\varepsilon')]. \end{aligned} \quad (6.5)$$

Since by using the inequality $\hbar\Omega_{\mathbf{q}} \ll T$ the product of the distribution functions can be approximated as

$$[1 - F^{(0)}(\varepsilon')] F^{(0)}(\varepsilon) [F^{(0)}(\varepsilon) - F^{(0)}(\varepsilon')] \simeq \frac{\hbar\Omega_{\mathbf{q}}}{T} [(1 - F^{(0)}(\varepsilon)) F^{(0)}(\varepsilon)]^2, \quad (6.6)$$

we see that

$$\frac{dU_{\text{ph}}}{dt} \simeq \left(\frac{s p_F}{T} \right)^2 V^2 \Delta G. \quad (6.7)$$

As the factor $s p_F / T$ is small we come to the conclusion that the mechanical energy dissipated within the wire is much greater than the energy transferred to the phonon system. Actually this is the case for which one can discard the terms $\hbar\Omega_{\mathbf{q}}$ in the δ -functions in equation (3.14). This statement is valid both for calculation of ΔG and of the heat release within the nanowire.

One should keep in mind, however, that actually the electron–phonon collisions are inelastic. To derive the Boltzmann equation for the conduction electrons of the nanowire as well as the equation for the entropy production one should take into account the inelasticity. Thus the quasimomentum relaxation is in the present case accompanied by energy relaxation. The latter, however slow, is nevertheless essential for the applicability of our methods of the heat release calculation. (It is worthwhile noting that the phase coherence of the electron wavefunctions is destroyed as a result of such inelasticity.)

We could have easily considered the contribution of all the relevant transitions of the type $n \longleftrightarrow n'$. We would get for q_z the estimate $q_z \lesssim p_F / \hbar$. As for the matrix element $|\langle n | \exp(i\mathbf{q}\perp \mathbf{r}\perp) | n' \rangle|$, we have the same upper estimate for it as above. As a result, the estimate (6.7) would not change.

7. Conclusion

The methods outlined in the present paper can be generalized to consider more complicated arrangements of the circuits. To know the rate of heat generation may be important for estimates concerning various devices in which the structure of the circuits is more involved.

Reference [17] and the present paper also lay the foundation for treatment of non-ohmic effects. They should be observable for $eV \gtrsim T$. In the present paper we consider a linear case where eV much smaller than T , and we assume that the length of the nanostructure is much smaller than the electron mean free path. A nonlinear case would be physically much richer. In this case one should consider the nonequilibrium electron and phonon system on an equal footing: there could be phonon accumulation in the vicinity of the wire and even many-phonon processes could be of importance. Probably all these processes might take place if the inelastic electron–phonon scattering length is comparable to the length of the wire and if the applied

voltage is big enough. However, in the present paper we restrict ourselves to much more simple linear case. In the classical regime, nonlinear phenomena in the current–voltage characteristics of point contacts between normal metals were observed and discussed in a pioneering work by Yanson [25].

In summary, we have discussed the Joule heat generation for the phonon-assisted ohmic resistance. The heat is released not only in the reservoirs but also in the course of electron–phonon collisions in the quantum wire. We believe that this conclusion can be verified by the technique used in [12]. A quantum point contact can be fabricated by joining the interior of a quantum wire and electron gas outside the wire using additional gates (see [12] where much more complicated nanostructures have been defined and employed). We expect that due to the Joule heat released in the interior of the nanostructure there would be a heat flow through such a quantum point contact that could be measured by the methods used in [12].

We have demonstrated that to calculate the Joule heat generation with regard to the electron collisions it is sufficient (i) to treat the kinetic part of the problem up to the first order in the voltage drop across the nanostructure (or in electric field \mathbf{E}) and (ii) to disregard the energy exchange with the phonons. The total rate of heat generation in each reservoir is the same. This is a consequence of an electron–hole symmetry present in the case of a strong Fermi degeneracy.

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